

ROTORDYNAMIC COEFFICIENTS FOR LABYRINTH SEALS CALCULATED BY
MEANS OF A FINITE DIFFERENCE TECHNIQUE

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The compressible, turbulent, time dependent and three dimensional flow in a labyrinth seal can be described by the Navier-Stokes equations in conjunction with a turbulence model. Additionally, equations for mass and energy conservation and an equation of state are required. To solve these equations, a perturbation analysis is performed yielding zeroth order equations for centric shaft position and first order equations describing the flow field for small motions around the seal center. For numerical solution a finite difference method is applied to the zeroth and first order equations resulting in leakage and dynamic seal coefficients respectively.

SYMBOLS

F_1, F_2	Forces on the shaft in z, y direction
U_1, U_2	shaft displacements
K, k	direct and cross-coupled stiffness
C, c	direct and cross-coupled damping
u, v, w	axial, radial and circumferential velocity
p, k, ϵ	pressure, turbulence energy, turbulent energy dissipation
T	temperature
μ_1, μ_t, μ_e	laminar, turbulent and effective viscosity ($\mu_e = \mu_1 + \mu_t$)
ρ	density
c_p	specific heat for constant pressure
t	time
z, r, θ	axial, radial and circumferential coordinate
η	radial coordinate after transformation
Pr	turbulent Prandtl number
$\sigma_k, \sigma_\epsilon$	constants of the k- ϵ model
C_μ, C_1, C_2	constants of the k- ϵ model
G	production term in turbulence model
$Diss$	laminar dissipation
ϕ	general variable
S_ϕ	general source term
C_o	seal clearance for centric shaft position
δ	seal clearance for eccentric shaft position
r_o	radius of the precession motion of the shaft

$e = r_o / C_o$	perturbation parameter
ω	rotational frequency of the shaft
Ω	precession frequency of the shaft
r_i	radius from seal center to end of seal fin
r_a	radius from seal center to stator
D_{1-10}	Transformation constants

SUBSCRIPTS

O	zeroth order variables
1	first order variables
R	Rotor
S	Stator
c	cosine
s	sine

INTRODUCTION

The problem of subsynchronous vibrations in high performance turbomachinery has been investigated for many years. However, for some destabilizing effects there is no satisfactory solution from the qualitative as well as quantitative points of view. In machines like turbocompressors and turbines, gas seals and especially labyrinths have an important influence on the dynamic behavior. These elements with turbulent flow conditions have the potential to develop significant forces

$$-\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} K & k \\ -k & K \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} + \begin{bmatrix} C & c \\ -c & C \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} \quad (1)$$

which may lead to self-excited vibrations of the shaft.

To predict the stability behavior of a high performance rotating machinery, the rotordynamic coefficients (eq. 1) of the turbulent seals have to be known. Many theoretical studies, numerical calculations and measurements have been carried out to determine the stiffness and damping characteristics. Most of the recent theoretical developments are based on a bulk flow theory, considering the shear stresses only at the wall but not within the fluid. Nelson (ref. 1) has derived a procedure with a 'one volume' model for straight and tapered seals. Iwatsubo (ref. 2) as well as Childs (ref. 3) have also used the one volume model for labyrinth seals. However, in case of such complicated seal geometries difficulties may occur, when neglecting the local turbulence characteristics within the fluid, particularly at locations with strong velocity changes.

Wyssmann et al. and Wyssmann (ref. 4 and 5) have extended the bulk flow theory for labyrinth seals, working with a two volume model. The latter takes into account the shear stresses between the core flow and the jet flow. Wyssmann's two volume model is derived from results of a finite difference calculation for a rotationally symmetric single cavity turbulent flow (centered position of the shaft), based on the time averaged Navier-Stokes equations with a $k-\epsilon$ model. A first attempt to apply finite difference methods to labyrinth seals was made by Rhode et.al. (ref. 6), who

calculated only one chamber seals for centric shaft position. However, bulk flow theories show a substantial lack in the prediction of labyrinth seal coefficients. As already shown by Nordmann, Dietzen and Weiser (ref. 7), the solution of the time-averaged conservation equations for momentum, mass and energy in conjunction with a k-ε turbulence model by means of a finite difference technique is a physically more realistic way for calculating rotordynamic coefficients of gas seals.

This paper is concerned with the extension of the already mentioned calculation procedure for annular seals to straight-through labyrinths.

GOVERNING EQUATIONS

To describe the flow in a labyrinth seal, we use the time-averaged conservation equations for momentum, mass and energy and the equation of state for a perfect gas. The correlation terms of the turbulent fluctuation quantities are modelled via the k-ε turbulence model of Launder and Spalding (ref. 8).

All these differential equations can be arranged in the following generalized form:

$$\begin{aligned} \frac{\partial}{\partial t}(\rho\phi) + \frac{1}{r} \frac{\partial}{\partial r}(\rho r v \phi) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho w \phi) + \frac{\partial}{\partial x}(\rho u \phi) = \\ \frac{1}{r} \frac{\partial}{\partial r}(r \Gamma \frac{\partial \phi}{\partial r}) + \frac{1}{r} \frac{\partial}{\partial \theta}(\Gamma \frac{1}{r} \frac{\partial \phi}{\partial \theta}) + \frac{\partial}{\partial x}(\Gamma \frac{\partial \phi}{\partial x}) + S_\phi \end{aligned} \quad (2)$$

where ϕ stands for any of the dependent variables, and the corresponding values of $\Gamma\phi$ and S_ϕ are indicated in Table 1 (see Appendix A).

PERTURBATION ANALYSIS

Obviously, this set of equations is not solvable due to the limited performance capabilities of today's computers. Therefore, a stepwise procedure is performed in order to reduce the three dimensional and time dependent flow problem to a two dimensional time independent one.

First, the governing equations are transformed to another coordinate system by introducing a new radial coordinate η whereby the eccentrically moving shaft is converted to a shaft rotating in the center of the seal (Fig. 1).

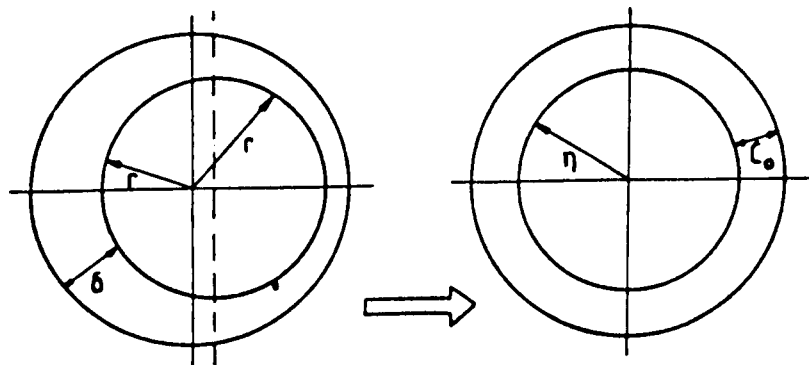
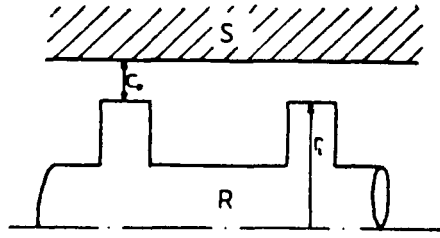


Fig. 1: Coordinate Transformation $\eta = \eta(r, t, \theta)$

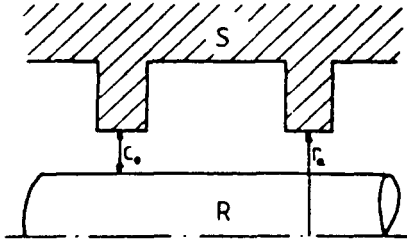
The transform functions are given in Fig. 2.



Strips on rotor:

$$\eta = r_1 + \frac{C_o \delta (r - r_1)}{(C_o - \delta)(r - r_1) + C_o \delta}$$

(3)



Strips on stator:

$$\eta = r_a + \frac{C_o \delta (r_a - r)}{(C_o - \delta)(r_a - r) - C_o \delta}$$

Fig. 2: Transform functions

Assuming small shaft motions on a circular orbit around the centric position we perform a perturbation analysis by introducing the following expressions for the dependent variables into the governing equations:

$$\begin{aligned} u &= u_o + e u_1 & v &= v_o + e v_1 & w &= w_o + e w_1 \\ p &= p_o + e p_1 & T &= T_o + e T_1 & \rho &= \rho_o + e \rho_1 \end{aligned} \quad (4)$$

This procedure yields a set of zeroth order equations governing the centric flow field and a set of first order equations describing the flow field for small eccentric shaft motions.

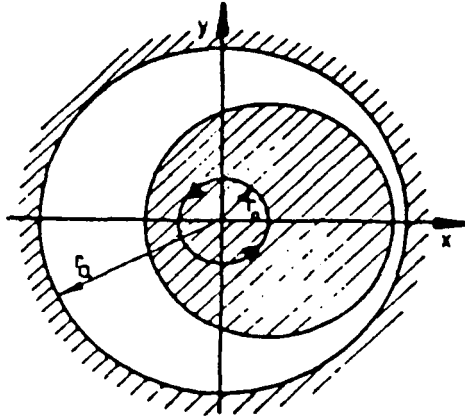
The first order equations depend also on the circumferential coordinate θ and on time t . To eliminate the circumferential derivations, we insert circumferentially periodic solutions for the first order variables in the corresponding set of equations:

$$\begin{aligned} u_1 &= u_{1c} \cos \theta + u_{1s} \sin \theta & v_1 &= v_{1c} \cos \theta + v_{1s} \sin \theta \\ w_1 &= w_{1c} \cos \theta + w_{1s} \sin \theta & p_1 &= p_{1c} \cos \theta + p_{1s} \sin \theta \\ T_1 &= T_{1c} \cos \theta + T_{1s} \sin \theta & \rho_1 &= \rho_{1c} \cos \theta + \rho_{1s} \sin \theta \end{aligned} \quad (5)$$

After separating the equations into sine and cosine terms and rearranging them by introducing the following complex variables,

$$\begin{aligned}
\bar{u}_1 &= u_{1c} + i u_{1s} & \bar{v}_1 &= v_{1c} + i v_{1s} & \bar{w}_1 &= w_{1c} + i w_{1s} \\
\bar{p}_1 &= p_{1c} + i p_{1s} & \bar{T}_1 &= T_{1c} + i T_{1s} & \bar{\rho}_1 &= \rho_{1c} + i \rho_{1s}
\end{aligned} \tag{6}$$

a circular shaft precession orbit (Fig. 3) with frequency Ω and corresponding solutions for the first order variables are assumed whereby the temporal derivations can be eliminated:



$$\begin{aligned}
\bar{h}_1 &= r_0 e^{i\Omega t} \\
&\text{(change in clearance)}
\end{aligned}$$

Fig. 3: Circular shaft orbit

$$\begin{aligned}
\bar{u}_1 &= \hat{u}_1 e^{i\Omega t} & \bar{v}_1 &= \hat{v}_1 e^{i\Omega t} & \bar{w}_1 &= \hat{w}_1 e^{i\Omega t} \\
\bar{p}_1 &= \hat{p}_1 e^{i\Omega t} & \bar{T}_1 &= \hat{T}_1 e^{i\Omega t} & \bar{\rho}_1 &= \hat{\rho}_1 e^{i\Omega t}
\end{aligned} \tag{7}$$

Finally the zeroth and first order equations are arranged to a generalized form:

$$\frac{\partial}{\partial x}(\rho_o u_o \phi) - \frac{\partial}{\partial x}(\Gamma_\phi \frac{\partial \phi}{\partial x}) + \frac{1}{\eta} \frac{\partial}{\partial \eta}(\eta \rho_o v_o \phi) - \frac{1}{\eta} \frac{\partial}{\partial \eta}(\Gamma_\phi \eta \frac{\partial \phi}{\partial \eta}) = S_\phi \tag{8}$$

Table 2 (see Appendix B) indicates the values for the zeroth and first order equations.

The constants D_1 - D_{10} (given in the Appendix C) arise from the coordinate transformation and depend only on zeroth order variables. These terms are zero for the labyrinth chambers.

FINITE DIFFERENCE METHOD

For the numerical solution of the governing equations we apply a finite difference technique in the same manner as suggested in many publications (see e.g. ref. 9). First the calculation domain is covered by a finite difference grid (Fig. 4). Discretizing the equations results in algebraic expressions linking every node point to his four neighbours. Solution of zeroth order equations results in flow field, pressure distribution, and leakage. For first order solution, the equations for the turbulence quantities k , ϵ can be dropped due to assumed small rotor motions. Because Eq. (1) contains four unknown parameters (K, k, C, c), first order solution is performed for two different shaft precession frequencies Ω . This results in two sets of dynamic forces F_1 , F_2 obtained from the integration of the pressure perturbations (see Eq. 9).

$$\begin{aligned}
 -\frac{F_1}{r_o} &= \frac{\pi r_i}{C_o} \int_L p_{1c} dx \\
 -\frac{F_2}{r_o} &= \frac{\pi r_i}{C_o} \int_L p_{1s} dx
 \end{aligned} \tag{9}$$

Finally the dynamic coefficients are calculated from Eq. 10.

$$\begin{aligned}
 -\frac{F_1}{r_o} &= K + c\Omega \\
 -\frac{F_2}{r_o} &= -k + C\Omega
 \end{aligned} \tag{10}$$

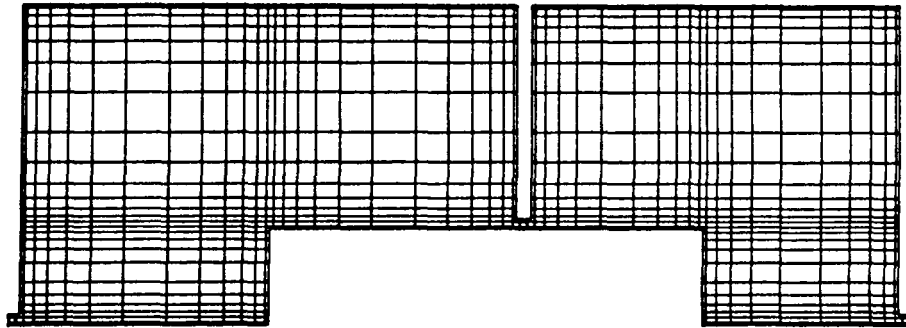


Fig. 4: Nonuniform finite difference grid for labyrinths

BOUNDARY CONDITIONS

- Zeroth order equations

For the velocities the near wall regions are represented by the logarithmic wall law. The entrance conditions are iteratively calculated depending on entrance Mach number and loss coefficient. The entrance swirl is assumed to be known.

- First order equations

Due to shaft precession and coordinate transformation, one has to pay attention to the following boundary conditions for the radial and circumferential velocities:

$$\begin{aligned}
 v_{1S} &= (0..0.) & \hat{w}_{1S} &= (0..0.) \\
 \hat{v}_{1R} &= (0..(\Omega-\omega)c_o) & \hat{w}_{1R} &= (\Omega c_o, 0.)
 \end{aligned}$$

For the pressure, the exit perturbation is zero in case of exit Mach number less than one ("undercritical" flow) which occurred in the application examples.

APPLICATION

For testing our program, we made comparative calculations to measurements performed by Benckert (ref. 10).

Seal Data:

Shaft diameter:	0,150 m
Strip height :	5,5 mm
Strip pitch :	8 mm
Clearance :	0,5 mm
Number of chambers :	3
Entrance pressure p_o :	1,425 bar
Exit pressure p_a :	0,950 bar
Reservoir temperature:	300 K
Laminar viscosity :	$1,8 \cdot 10^{-5}$ Pa s
Perfect gas constant :	287,06 J/kgK
Specific heat ratio :	1,4
Rotor surface velocity:	0 m/s

The calculation grid, the flow field and the pressure distribution are shown in Fig. 5-7.

In Fig. 8-10 mass flow, direct and crosscoupled stiffness are plotted versus the swirl parameter, as defined by Benckert ($E_o = 0,5 \cdot \rho_o \cdot w_o^2 / (p_o - p_a)$).

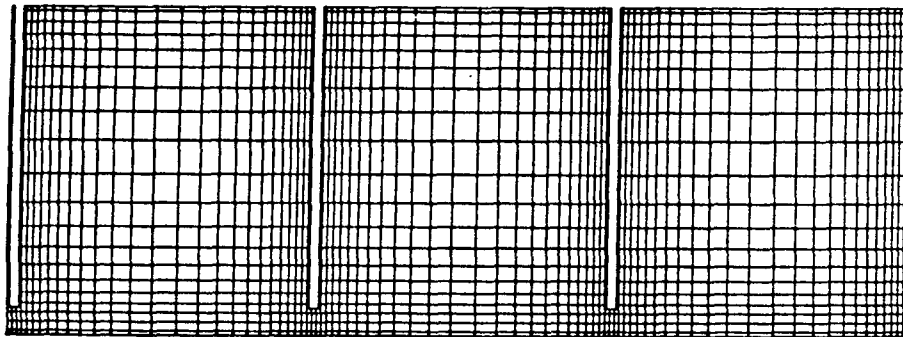


Fig. 5: Finite difference grid

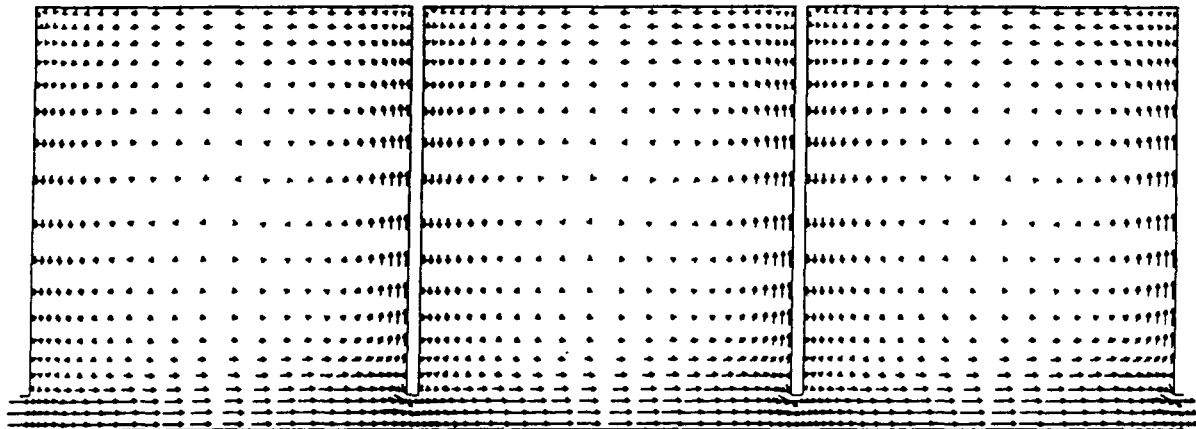


Fig. 6: Flow field

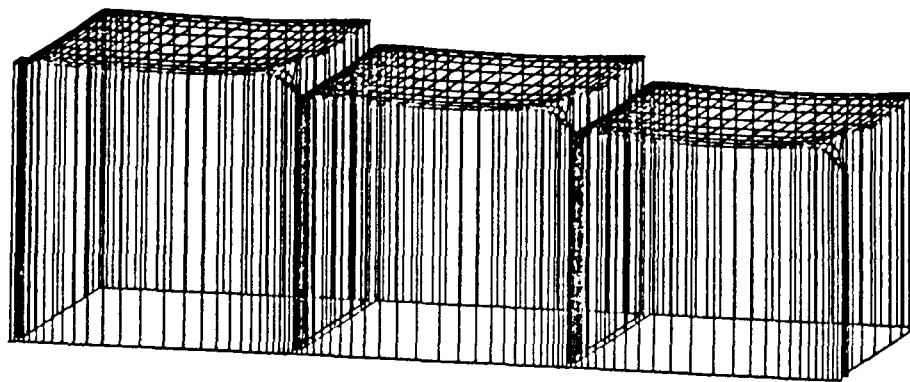


Fig. 7: Pressure distribution

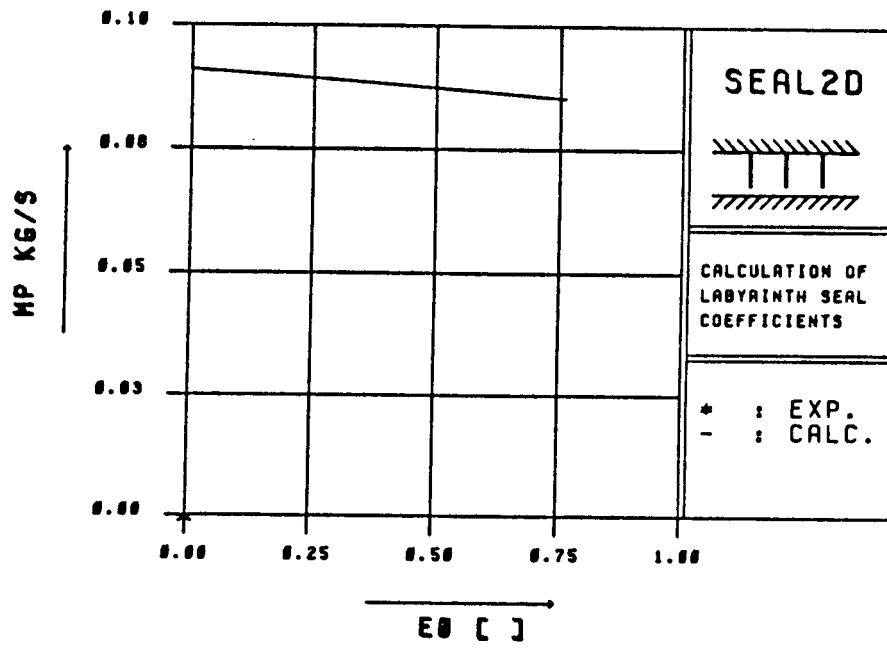


Fig. 8: Mass flow

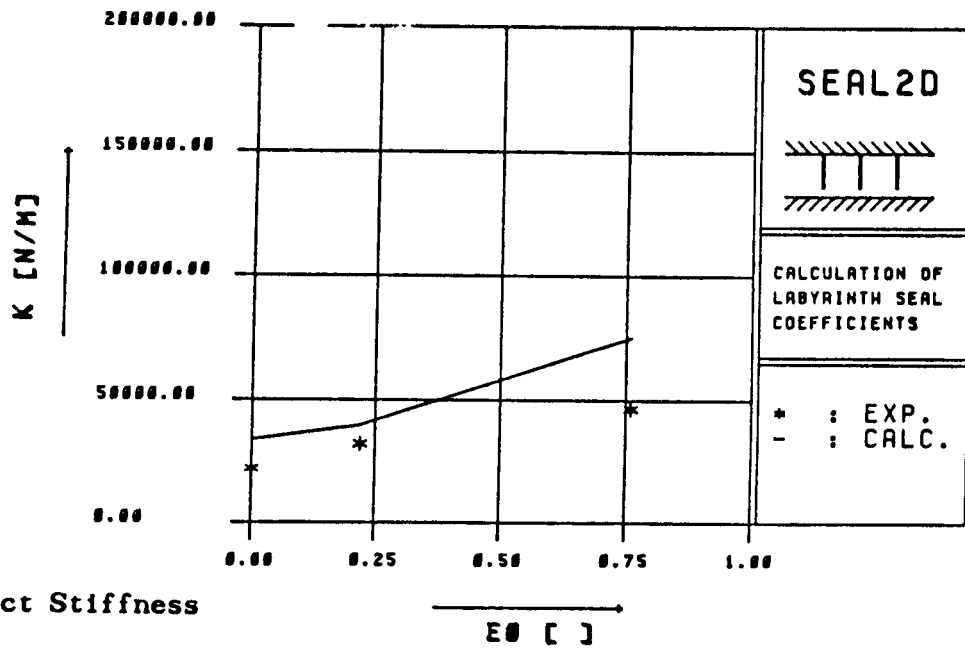


Fig. 9: Direct Stiffness

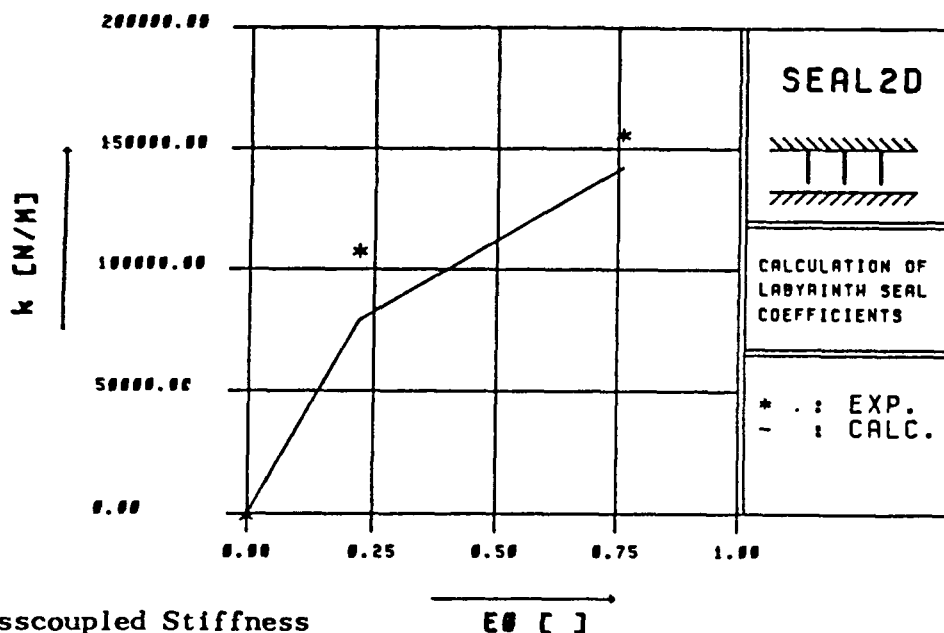


Fig. 10: Crosscoupled Stiffness

CONCLUSION

We have shown that a finite difference method based on the Navier-Stokes equations in conjunction with a turbulence model is more suitable to describe the dynamic coefficients of gas seals than the simpler models based on bulk flow theories. Of course the calculation time is also much greater. But great progress is still possible. For example, advanced solution algorithms like Multi-Grid, Newton-Schemes, SIP (strongly implicit procedure) can be implemented to speed up the computer program.

Further on, a three dimensional calculation procedure is currently under development to determine the coefficients of more complex geometries like stepped and interlocking labyrinths (Fig. 11,12).

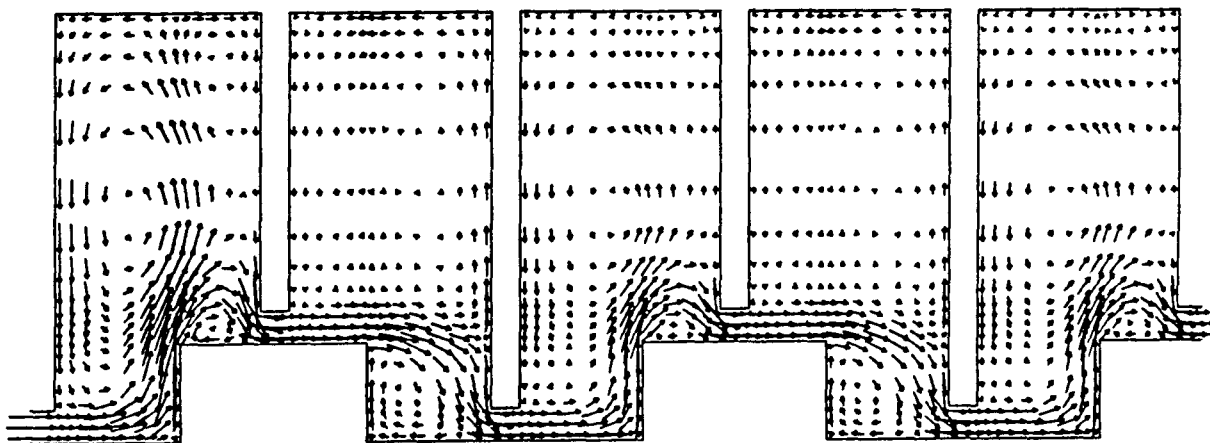


Fig. 11: Stepped seal

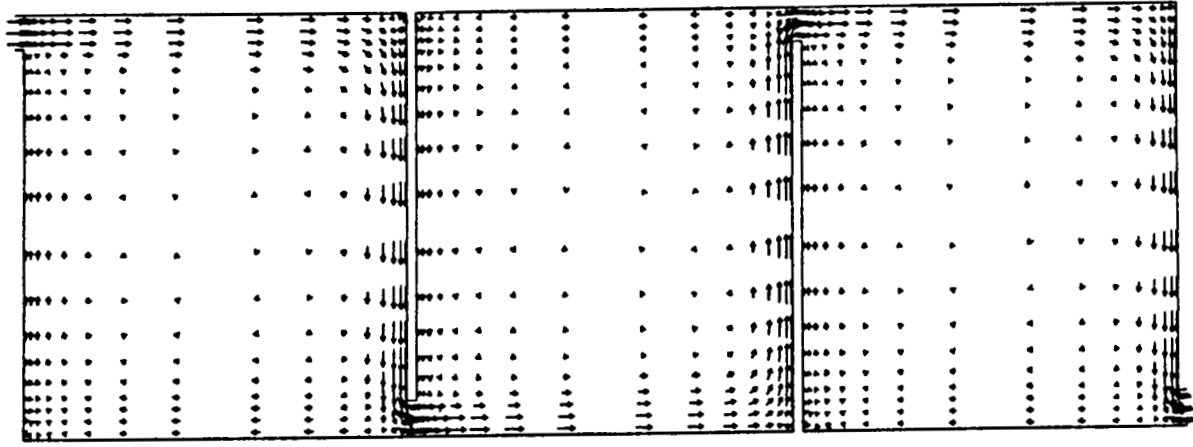


Fig. 12: Interlocking seal

APPENDIX A

Table 1: Governing equations of turbulent seal flow

ϕ	Γ_ϕ	S_ϕ
u	μ_e	$-\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}(\mu_e \frac{\partial u}{\partial x}) + \frac{1}{r} \frac{\partial}{\partial r}(r \mu_e \frac{\partial v}{\partial r}) + \frac{1}{r} \frac{\partial}{\partial \theta}(\mu_e \frac{\partial w}{\partial \theta})$
v	μ_e	$-\frac{\partial p}{\partial r} + \frac{\partial}{\partial x}(\mu_e \frac{\partial u}{\partial r}) + \frac{1}{r} \frac{\partial}{\partial r}(r \mu_e \frac{\partial v}{\partial r}) + \frac{1}{r} \frac{\partial}{\partial \theta}(r \mu_e \frac{\partial}{\partial r}(\frac{w}{r}))$ $-\frac{2}{r^2} \mu_e \frac{\partial w}{\partial \theta} - \frac{2}{r^2} \mu_e v + \frac{\rho}{r} w^2$
w	μ_e	$-\frac{1}{r} \frac{\partial p}{\partial \theta} + \frac{\partial}{\partial x}(\frac{1}{r} \mu_e \frac{\partial u}{\partial \theta}) + \frac{1}{r} \frac{\partial}{\partial r}(\mu_e \frac{\partial v}{\partial \theta}) + \frac{1}{r} \frac{\partial}{\partial \theta}(\frac{1}{r} \mu_e \frac{\partial w}{\partial \theta})$ $+ \frac{1}{r^2} \mu_e \frac{\partial v}{\partial \theta} - \frac{w}{r^2} \frac{\partial}{\partial r}(r \mu_e) + \frac{1}{r} \frac{\partial}{\partial \theta}(\frac{2}{r} \mu_e v) - \frac{\rho}{r} vw$
p	0	0
T	μ_e / Pr	$\frac{1}{c_p} (\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial r} + \frac{w}{r} \frac{\partial p}{\partial \theta}) + \frac{1}{c_p} (Diss + \rho \epsilon)$
k	μ_e / σ_k	$G - \rho \epsilon$
ϵ	μ_e / σ_ϵ	$C_1 \frac{\epsilon G}{k} - C_2 \rho \frac{\epsilon^2}{k}$

Equation of State for a perfect gas:

$$p = \rho RT$$

Constants of the k- ϵ model

$$C_\mu = 0.09 \quad C_1 = 1.45 \quad C_2 = 1.91 \quad \sigma_k = 1.0 \quad \sigma_\epsilon = 1.22 \quad Pr = 0.9$$

Production term in the equations for the turbulence quantities:

$$G = \mu_e \left[2 \left(\left(\frac{\partial v}{\partial r} \right)^2 + \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{1}{r} \frac{\partial w}{\partial \theta} + \frac{v}{r} \right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial r} \right)^2 + \left(\frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial r} - \frac{w}{r} \right)^2 \right. \right. \\ \left. \left. + \left(\frac{\partial w}{\partial x} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right)^2 - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial w}{\partial \theta} + \frac{1}{r} \frac{\partial}{\partial r} (rv) \right)^2 \right] \right]$$

Laminar Dissipation in the energy equation:

$$Diss = \mu_l \left[2 \left(\left(\frac{\partial v}{\partial r} \right)^2 + \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{1}{r} \frac{\partial w}{\partial \theta} + \frac{v}{r} \right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial r} \right)^2 + \left(\frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial r} - \frac{w}{r} \right)^2 \right. \right. \\ \left. \left. + \left(\frac{\partial w}{\partial x} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right)^2 - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial w}{\partial \theta} + \frac{1}{r} \frac{\partial}{\partial r} (rv) \right)^2 \right] \right]$$

APPENDIX B

Table 2a: Zeroth order equations

ϕ	Γ_ϕ	S_ϕ
u_o	μ_e	$-\frac{\partial p_o}{\partial x} + \frac{\partial}{\partial x} \left(\mu_e \frac{\partial u_o}{\partial x} \right) + \frac{1}{\eta} \frac{\partial}{\partial \eta} \left(\mu_e \eta \frac{\partial v_o}{\partial x} \right)$
v_o	μ_e	$-\frac{\partial p_o}{\partial \eta} + \frac{\partial}{\partial x} \left(\mu_e \frac{\partial u_o}{\partial \eta} \right) + \frac{1}{\eta} \frac{\partial}{\partial \eta} \left(\mu_e \eta \frac{\partial v_o}{\partial \eta} \right) - \frac{2}{\eta} \mu_e v_o + \frac{\rho_o}{\eta} w_o^2$
1	0	0
w_o	μ_e	$-\frac{w_o}{\eta} \frac{\partial}{\partial \eta} (\eta \mu_e) - \frac{\rho_o}{\eta} v_o w_o$
k_o	$\frac{\mu_e}{\sigma_k}$	$G - \rho_o \epsilon$
ϵ_o	$\frac{\mu_e}{\sigma_\epsilon}$	$C_1 \frac{\epsilon}{k} G - C_2 \rho_o \frac{\epsilon^2}{k}$
T_o	$\frac{\mu_e}{Pr}$	$\frac{1}{c_p} \left(u_o \frac{\partial p_o}{\partial x} + v_o \frac{\partial p_o}{\partial \eta} \right) + \frac{1}{c_p} (Diss + \rho_o \epsilon)$

Equation of State: $\rho_o = \frac{p_o}{R \cdot T_o}$

Table 2b: First order equations

Equation of State: $\hat{\rho}_1 = \rho_o \left[\frac{\hat{p}_1}{p_o} - \frac{\hat{T}_1}{T_o} \right]$

ϕ	Γ_ϕ	S_ϕ
\hat{u}_1	μ_e	$-\frac{\partial \hat{p}_1}{\partial x} \frac{\partial}{\partial x} (\mu_e \frac{\partial \hat{u}_1}{\partial x}) + \frac{1}{\eta} \frac{\partial}{\partial x} (\eta \mu_e \frac{\partial \hat{v}_1}{\partial x}) - \frac{\partial}{\partial x} (\rho_o u_o \hat{u}_1)$ $- \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta \rho_o u_o \hat{v}_1) - \frac{\mu_e}{\eta} \hat{u}_1 + i \rho_o (\frac{w_o}{\eta} \Omega) u_1 + i \frac{\rho_o}{\eta} u_o \hat{w}_1 - i \frac{\mu_e \partial \hat{w}_1}{\eta \partial x}$ $- \frac{\partial}{\partial x} (u_o u_o \hat{\rho}_1) - \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta v_o u_o \hat{\rho}_1) + i \frac{u_o w_o}{\eta} \hat{\rho}_1 - i \Omega u_o \hat{\rho}_1 + D_1 + i D_2$
\hat{v}_1	μ_e	$-\frac{\partial \hat{p}_1}{\partial \eta} \frac{\partial}{\partial x} (\mu_e \frac{\partial \hat{u}_1}{\partial \eta}) + \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta \mu_e \frac{\partial \hat{v}_1}{\partial \eta}) - \frac{\partial}{\partial x} (\rho_o v_o \hat{u}_1)$ $- \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta \rho_o v_o \hat{v}_1) - i \mu_e \frac{\partial}{\partial \eta} (\frac{\hat{w}_1}{\eta}) - 3 \frac{\mu_e \hat{v}_1}{\eta^2}$ $+ i \rho_o (\frac{w_o}{\eta} - \Omega) \hat{v}_1 + (2 \frac{\rho_o w_o}{\eta} + i 2 \frac{\mu_e}{\eta^2} + i \frac{\rho_o v_o}{\eta}) \hat{w}_1$ $- \frac{\partial}{\partial x} (u_o v_o \hat{\rho}_1) - \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta v_o v_o \hat{\rho}_1) + \frac{w_o^2}{\eta} \hat{\rho}_1 + i \frac{v_o w_o}{\eta} \rho_1$ $- i \Omega v_o \hat{\rho}_1 + D_3 + i D_4$
\hat{w}_1	$\hat{\mu}_e$	$i \frac{\hat{p}_1}{\eta} - i \frac{\partial}{\partial x} (\frac{\mu_e \hat{u}_1}{\eta}) - i \frac{1}{\eta} \frac{\partial}{\partial \eta} (\mu_e \hat{v}_1) - \frac{\partial}{\partial x} (\rho_o w_o \hat{u}_1)$ $- \frac{1}{2} \frac{\partial}{\partial \eta} (\rho_o \eta^2 w_o \hat{v}_1) - 2 \frac{\mu_e \hat{w}_1}{\eta^2} - i 3 \frac{\mu_e \hat{v}_1}{\eta}$ $- (\frac{\rho_o v_o}{\eta} + i \rho_o \Omega + \frac{1}{2} \frac{\partial \eta \mu_e}{\partial \eta} - i 2 \frac{\rho_o w_o}{\eta}) \hat{w}_1$ $\frac{\partial}{\partial x} (u_o w_o \hat{\rho}_1) - \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta v_o w_o \hat{\rho}_1) - \frac{v_o w_o}{\eta} \hat{\rho}_1 + i \frac{w_o w_o}{\eta} \hat{\rho}_1$ $- i w_o \hat{\rho}_1 + D_5 + i D_6$
T_1	$\frac{\mu_e}{Pr}$	$\frac{1}{c_p} (u \frac{\partial \hat{p}_1}{\partial x} + \hat{u} \frac{\partial p_o}{\partial x} + v \frac{\partial p_o}{\partial \eta} + v \frac{\partial \hat{p}_1}{\partial \eta} - i \frac{w_o \hat{p}_1}{\eta}) - \frac{\mu_e}{Pr} \frac{1}{\eta} \hat{T}_1$ $- \frac{\partial}{\partial x} (\rho_o T_o \hat{u}_1) - \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta \rho_o T_o \hat{v}_1) + \frac{i}{\eta} (\rho_o T_o \hat{w}_1 + \rho_o w_o \hat{T}_1)$ $+ i \Omega (\frac{\hat{p}_1}{c_p} - \rho_o \hat{T}_1) - \frac{\partial}{\partial x} (u_o T_o \hat{\rho}_1) - \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta v_o T_o \hat{\rho}_1)$ $+ i \frac{T_o w_o}{\eta} \hat{\rho}_1 - i \Omega T_o \hat{\rho}_1 + \frac{\epsilon}{c_p} \hat{\rho}_1 + D_7 + i D_8$

Only the first order continuity-equation to determine \hat{p}_1 shows a slightly modified form.

$$\begin{aligned} \frac{\partial}{\partial x}(\rho_o \hat{u}_1) + \frac{1}{\eta} \frac{\partial}{\partial \eta}(\eta \rho_o \hat{v}_1) &= i \frac{\rho_o}{\eta} \hat{w}_1 - \frac{\partial}{\partial x}(u_o \hat{p}_1) - \frac{1}{\eta} \frac{\partial}{\partial \eta}(\eta v_o \hat{p}_1) \\ &+ i \frac{w_o}{\eta} \hat{p}_1 - i \Omega \hat{p}_1 + D_9 + i D_{10} \end{aligned}$$

APPENDIX C

First order transformation constants

Note: $r_x = r_a$ for strips on stator

$r_x = r_i$ for strips on rotor

$$\begin{aligned} D_1 &= \frac{2(r_x - \eta)}{c_o} \frac{\partial}{\partial \eta} (\mu_{eo} \frac{\partial u_o}{\partial \eta} + \mu_{eo} \frac{\partial v_o}{\partial x} - u_o v_o \rho_o) \\ &- \frac{(r_x - \eta)^2}{c_o \eta^2} (\mu_{eo} \frac{\partial u_o}{\partial \eta} + \mu_{eo} \frac{\partial v_o}{\partial x} - u_o v_o \rho_o) \end{aligned}$$

$$\begin{aligned} D_2 &= \frac{(r_x - \eta)^2}{c_o \eta} \frac{\partial}{\partial \eta} (\mu_{eo} \frac{\partial w_o}{\partial x} - \rho_o u_o w_o) \\ &+ \Omega \frac{(r_x - \eta)^2}{c_o} \frac{\partial}{\partial \eta} (\rho_o u_o) \end{aligned}$$

$$\begin{aligned} D_3 &= \frac{2(r_x - \eta)}{c_o} \frac{\partial}{\partial \eta} (2\mu_{eo} \frac{\partial v_o}{\partial \eta} - \rho_o v_o v_o) \\ &- \frac{(r_x - \eta)^2}{c_o \eta^2} (2\mu_{eo} \frac{\partial v_o}{\partial \eta} + \rho_o w_o w_o - \rho_o v_o v_o - 2\mu_{eo} \frac{v_o}{\eta}) \\ &- \frac{2(r_x - \eta)}{c_o} \frac{\partial p_o}{\partial \eta} \end{aligned}$$

$$\begin{aligned} D_4 &= \frac{(r_x - \eta)^2}{c_o \eta} \frac{\partial}{\partial \eta} (\mu_{eo} \eta \frac{\partial}{\partial \eta} (\frac{w_o}{\eta}) - \rho_o v_o w_o) \\ &+ \Omega \frac{(r_x - \eta)^2}{c_o} \frac{\partial}{\partial \eta} (\rho_o v_o) \end{aligned}$$

$$\begin{aligned} D_5 &= \frac{2(r_x - \eta)^2}{c_o \eta^2} (\rho_o v_o w_o - \mu_{eo} \eta \frac{\partial}{\partial \eta} (\frac{w_o}{\eta})) \\ &- \frac{2(r_x - \eta)}{c_o} \frac{\partial}{\partial \eta} (\rho_o v_o w_o - \mu_{eo} \eta \frac{\partial}{\partial \eta} (\frac{w_o}{\eta})) \end{aligned}$$

$$\begin{aligned} D_6 &= \frac{(r_x - \eta)^2}{c_o \eta} \frac{\partial}{\partial \eta} (2\mu_{eo} \frac{v_o}{\eta}) - \frac{\partial}{\partial \eta} (\rho_o w_o w_o - \frac{\partial p_o}{\partial \eta}) \\ &+ \Omega \frac{(r_x - \eta)^2}{c_o} \frac{\partial}{\partial \eta} (\rho_o w_o) \end{aligned}$$

$$\begin{aligned} D_7 &= \frac{(r_x - \eta)^2}{c_o \eta^2} (\rho_o v_o T_o - \frac{\mu_{eo}}{Pr} \frac{\partial T_o}{\partial \eta}) \\ &+ \frac{2(r_x - \eta)}{c_o} \frac{\partial}{\partial \eta} (\frac{\mu_{eo}}{Pr} \frac{\partial T_o}{\partial \eta} - \rho_o v_o T_o) \\ &+ \frac{1}{c_p} \frac{2(r_x - \eta)}{c_o} v_o \frac{\partial p_o}{\partial \eta} \end{aligned}$$

$$\begin{aligned} D_8 &= \frac{1}{c_p} \frac{w_o}{\eta} \frac{(r_x - \eta)^2}{c_o} \frac{\partial p_o}{\partial \eta} \\ &- \frac{(r_x - \eta)^2}{\eta c_o} \frac{\partial}{\partial \eta} (\rho_o w_o T_o) \\ &+ \Omega \frac{(r_x - \eta)^2}{c_o} \frac{\partial}{\partial \eta} ((\rho_o T_o) - \frac{p_o}{c_p}) \end{aligned}$$

$$D_9 = \frac{(r_x - \eta)^2}{c_o \eta^2} \rho_o v_o - \frac{2(r_x - \eta)}{c_o} \frac{\partial}{\partial \eta} (\rho_o v_o)$$

$$\begin{aligned} D_{10} &= \Omega \frac{(r_x - \eta)^2}{c_o} \frac{\partial \rho_o}{\partial \eta} \\ &- \frac{(r_x - \eta)^2}{c_o \eta} \frac{\partial}{\partial \eta} (\rho_o w_o) \end{aligned}$$

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